LETTER COMBINATORICS: A THEORY OF COUNTING PROBLEM.

(Marriage Problem)

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This is a teaching-fellow in its first publication at the above institution. Any omitting or errors are mine only and can be communicated to when needed.

ABSTRACT

Letter combinatorics is about sentences or phrases and counting problems. It is logical structured and involves discrete operations like subtraction, addition and multiplication. It is about alphanumeric labeling of sentences or phrases and proofing of combinatorial enumerations. This teaches the theory of combinatorics of sentences or phrases or words called Letter Combinatorics (*LC*) with the 8-bulletin requirements. A Marriage Problem(*MP*) made up of 5 sentences are used in the exploit of letter combinatorics.

A generating function is calculated for MP to handle constraints of arrangement/selection and the combinatorial enumerations of MP are also calculated. This example of letter combinatorics shows the calculation of *addition* , *multiplication* and *subtraction* principles of MP.

1.0 INTRODUCTION

1.Letter Combinatorics has or must have the following requirements:

- A countable number of sentences or phrases.
- Counting the size of selected phrases for likely occurrence of subset equality of letters is called *count*.
- A sentence with the number of letters specified is called Π.
- The logical structure of arithmetic such as +, - and = should be applied.
- Discrete operations must include count, addition, subtraction and sizes.
- The sizes of selected phrases are enumerated.
- Proofs with the discrete operations on

which the enumeration of sizes stops.

•There is a possibility of summation equal to Π.

A logical Structure is of the form:

$$L = \langle + | - \rangle =, \sum_{s}^{n}$$

$$, where\, n > 1$$

.

n is a sentence or phrase number and s under sum means sentential summation.

2.***(Count Equality Principle) Definition***: *The size of selected phrases for likely occurrence of subset equality of letters.*

An *r-combination* of *n* distinct objects is an unordered selection, or subset of *r* out of the *n* objects(letters). The fundamental skills of combinatorial reasoning on letter combinatorics is simply a class of counting problems. Count Equality is a principle of solution of specific

classes of counting problem with no arrangement but selection with repetition. The two main counting principles in the elements of classes of counting problem are addition and multiplication principles.

3.*Addition Principle* states that if there are r_1 different objects in the first set, r_2 different objects in the second set, ..., and r_m different objects in the *m*th set, and if the different sets are disjoint, then the number of ways to select an object from one of the *m* sets is $r_1 + r_2 + r_3 + r_4 \dots + r_m$. On the other hand, *Multiplication Principle* supposes a procedure can be broken into successive (ordered) stages with r_1 different outcomes in the first stage, r_2 different outcomes in the second stage, …, and r_m different outcomes in the *m*th stage. If the number of outcomes at each stage is independent of the choices in previous stages and if the composite outcomes are all distinct, the total procedure

has $r_1 \ x \ r_2 \ x \ r_3 \ x \ r_4 \ \ldots \ x \ r_m$ different composite outcomes.

4.A distribution problem is equivalent to an arrangement or selection problem with repetition. The focus on modeling distribution problems can be broken into sub-cases that can be counted in terms of simple permutation and combinations. The process of distributing *r* identical letter objects into *n* different letter objects is the selection equivalence of distribution.

Selection Equivalence of Letter Distribution[SELD] (Corollary 1): *The distributions of identical letter objects(word) are equivalent to letter selections.*

Thus, there $C(r+n-1, r) = \frac{(r+n-1, r)!}{r!(n-1)!} \equiv SELD$

__ $C(n-r)$ ***- Combination (Corollary 2)***: *Letter combinations are a general counting method of*

unordered outcome.

Distributing of distinct letter objects are equivalent to arrangements but letter combinations do not have that as a generally specialized distribution problem.

__ $n \equiv^{r}$ ***- Arrangement (Corollary 1):*** Letter combinations are not a general distribution problem of distinct letter objects.

__ $n \equiv^{r}$ ***- Arrangement (Corollary 2):*** Letter combinations are not a general arrangement of ordered outcome.

5.***(Letter Count Problem) Theorem***: *Letter combinatorics is a counting problem.*

6.(***Letter Cut) Lemma***: *A selected phrase is by cutting a number of possible letters.*

7.Finally, it is a counting problem and a selected phrase is by cutting a number of possible letters.

8.***(Cutting Strategy) Proposition***: *A strategy of cutting the possible matches can continue with as many comparisons as needed.*

9.***(Π-Sentence) Axiom***: *Π is a number of letters specified in a sentence.*

2.0 MARRIAGE PROBLEM (EXAMPLE)

(1) Damn it.

(2) What's wrong?

(3) It is a combination of 46 letters.

(4) Akua will not marry you.

(5) Pokua will not marry you.

10. *<u>Partition of Integers (Definition)</u>: A partition of countable integer, count to be a collection of positive integers whose sum is N.*

11. For the Marriage Problem(MP) Example, the partition of integers are:

<u>Damn</u> <u>it</u>. (1)

4 + 2 = N = 6.

<u>What's</u> <u>wrong</u> (2)

5 + 5 = N = 10.

The size of each of the sentences are in Appendix A.

12. The collection or set of a sum and the list of integers of the partition is in increasing order.

MP = { 6, 10 }, *MP* set is a set of marriage problem partitions of integers. The problem, MP 1 is equivalent to the number of integer solutions to

$$2e_2 + 4e_4 = |4+2| = 6 .$$

The generating function for the number(6: MP 1 case), is the ways that we can choose countable integers

$$\left(1+X^{2}+X^{4}+X^{6}\ldots\right)\left(1+X^{4}+X^{8}+X^{12}\ldots\right).$$

Generating functions are to handle constraints in selection and arrangement problems.

13. Combinatorial Enumeration of Letters

- $\sum_{s} N$ is the sentential summation for a sentence.

1. $\sum_{s}^{1} N$ Is the sentential summation for sentence 1

(From MP example): $\sum_{s}^{1} N = 6$.

2. $\sum_{s}^{2} N$ Is the sentential summation for sentence 2

(From MP example): $\sum_{s}^{2} N = 10$.

3. $\sum_{s}^{6} N$ Is the sentential summation for sentence 3

(From MP example): $\sum_{s}^{6} N = 27$.

4. $\sum_{s}^{4} N$ Is the sentential summation for sentence 4

(From MP example): $\sum_{s}^{4} N = 19$.

5. $\sum_{s}^{5} N$ Is the sentential summation for sentence 5

(From MP example): $\sum_{s}^{5} N = 20$.

MP={6 ,10, 27, 19, 20}

•***Π = 46.***

•*Logical Structure*

$$L= \left(+, \sum_{s}^{4}, \sum_{s}^{3} \right)$$

•*Discrete Operation (Proof)*

•The Equality Principle on the Marriage Problem is

$$\sum_{s}^{L} = \sum_{s}^{3} + \sum_{s}^{4}$$
$$= 19+27$$
$$= 46. \qquad =\boldsymbol{\mathit{\Pi}}$$

Π stops on the enumeration of size 46 with the sentential summation for sentence 4 and sentential summation for sentence 3.

•*Discrete Operation (Addition)*

The Addition principle on the Marriage Problem results:

$$\sum_{s}^{L_A} = \sum_{s}^{1} + \sum_{s}^{2} + \sum_{s}^{3} + \sum_{s}^{4} + \sum_{s}^{5}$$
$$= 6+ \ 10+ \ 27+ \ 19+ \ 20$$
$$= 82 .$$

•*Discrete Operation (Multiplication)*

The Multiplication principle on the Marriage Problem results :

$$\sum_{s}^{L_M} = \sum_{s}^{1} x \sum_{s}^{2} x \sum_{s}^{3} x \sum_{s}^{4} x \sum_{s}^{5}$$
$$= 6 \quad x \quad 10 \quad x \quad 27 \quad x \quad 19 \quad x \quad 20$$
$$= 615600$$

•*Discrete Operation (Subtraction)*

The Subtraction principle on the Marriage Problem results:

$$\sum_{s}^{L_S} = \sum_{s}^{5} - \sum_{s}^{4} - \sum_{s}^{3} - \sum_{s}^{2} - \sum_{s}^{1}$$
$$= 20 - \quad 19 - \quad 27 - \quad 10 - \quad 6$$
$$= -42$$

•Discrete Meta-Operation (Addition-Subtraction)

$\sum_{s}^{L_{AS}}$ is the meta-sentential summation.

$$\sum_{s}^{L_{AS}} = \left\{ \sum_{s}^{L_1}, \sum_{s_3}^{L_2}, \sum_{s_1}^{L_3}, \sum_{s_3}^{L_4}, \sum_{s_2}^{L_5} \right\}$$

- The real principles on the Marriage Problem are based on the maximum and minimum sentential

summations: minimum=6 and maximum=20. The MP(3) is not a real marriage problem so it is not considered in MPSet={6, 10, 19, 20}. A discrete subtraction operation on the real principles gives

$$\sum_{s}^{L_{rS}} = \sum_{s}^{4} - \sum_{s}^{1}$$
$$= 20 - 6$$
$$= 14$$

The new RMPSet={ {6, 10, 19, 20} U {14} }

={ 6, 10, 14, 19, 20}.

Discrete Operations on RMPSet:

Addition Principle:

$$\sum_{s}^{L_A} = \sum_{s}^{1} + \sum_{s}^{2} + \sum_{s}^{3} + \sum_{s}^{4} + \sum_{s}^{5}$$
$$= 6 + 10 + 14 + 19 + 20$$
$$= 69$$

Multiplication Principle:

$$\sum_{s}^{L_M} = \sum_{s}^{1} x \sum_{s}^{2} x \sum_{s}^{3} x \sum_{s}^{4} x \sum_{s}^{5}$$
$$= 6 \quad x \quad 10 \quad x \quad 14 \quad x \quad 19 \quad x \quad 20$$
$$= 319200$$

Subtraction Principle:

$$\sum_{s}^{L_S} = \sum_{s}^{5} - \sum_{s}^{4} - \sum_{s}^{3} - \sum_{s}^{2} - \sum_{s}^{1}$$
$$= 20 - \quad 19 - \quad 14 - \quad 10 - \quad 6$$
$$= -29$$

3.0 SUMMARY

This is letter combinatorics on a real marriage problem. This teaches what is all about letter combinatorics of words, sentences or phrases. SELD, which is Selections Equivalence of Letter Distribution is a corollary. Corollary 2 is about __ $C(n-r)$ ***that states that letter combinatorics is all about counting***

methods of unordered outcome. There are two arrangement corollaries after the combination corollary. The Equality Principle on Marriage Problems is calculated and resulted as 46 for Marriage Count Problems. The research finding is based on the introduction of discrete subtraction operation on the minimum and maximum summations in LC of RMP.

APPENDIX A- Size Graphics Illustration

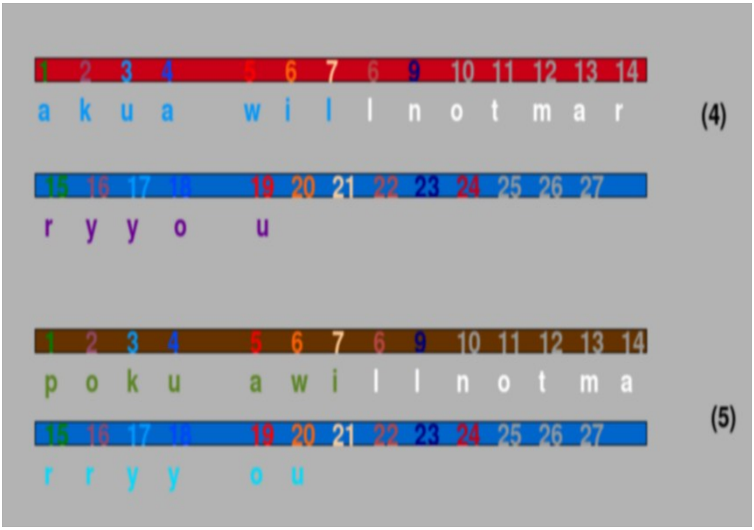
Illustration 1: Sentence (1) Size Graphics

Illustration 2: Sentence (2) Size Graphics

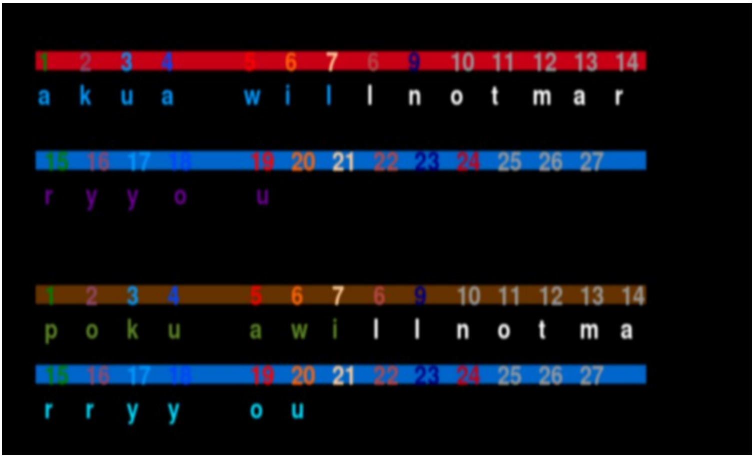
Illustration 3: Sentence (3) Size Graphics

Illustration 4: Sentence (4) Size Graphics

Illustration 5: Sentence (5) Size Graphics


1 2 3 4 5 6 7 8 9 10 11 12 13 14
a k u a w i l l n o t m a r
(4)
15 16 17 18 19 20 21 22 23 24 25 26 27
r y y o u
1 2 3 4 5 6 7 8 9 10 11 12 13 14
p o k u a w i l l n o t m a
(5)
15 16 17 18 19 20 21 22 23 24 25 26 27
r r y y o u


1 2 3 4 5 6
D a mn i t
1 2 3 4 5 6 7 8 9 10
W h a t ' s w r o n g
1 2 3 4 5 6 7 8 9 10 11 12 13 14
i t i s a c o m b i n a t i
15 16 17 18 19 20 21 22 23 24 25 26 27
o n o f 4 6 l e t t e r s


1 2 3 4 5 6 7 6 9 10 11 12 13 14
a k u a w i l l n o t m a r
15 16 17 18 19 20 21 22 23 24 25 26 27
r y y o u
1 2 3 4 5 6 7 6 9 10 11 12 13 14
p o k u a w i l l n o t m a
15 16 17 18 19 20 21 22 23 24 25 26 27
r r y y o u

APPENDIX B- Table on Size, Position, Sentence and Words.

Sentence	*Position*	Word	Size
1	1	**Damn**	4
1	2	**it**	2
2	1	**What's**	5
2	2	**wrong**	5
3	1	**it**	2
3	2	**is**	2
3	3	**a**	1
3	4	**combination**	11
3	5	**of**	2
3	6	**46**	2
3	7	**letters**	7
4	1	**Akua**	4
4	2	**will**	4
4	3	**not**	3

Sentence	*Position*	**Word**	Size
4	4	**marry**	5
4	5	**you**	3
5	1	**Pokua**	5
5	2	**will**	4
5	3	**not**	3
5	4	**marry**	5
5	5	**you**	3

Bibliography

1. *Alan Tucker(2012), Applied Combinatorics, Wiley, 6th Edition, ISBN:1118210115.*